Algebraic geometry, integrable systems and automorphic forms

May 26-30, 2025

Lille (France)

Laboratoire Paul Painlevé

Salle de réunions, bâtiment M2

Scientific Committee: Ekaterina Amerik, Valery Gritsenko, Anne Moreau, Pierre Vanhove

Organizers: Abdelghani El Mazouni,Fatima Laytimi, Dimitri Markushevich

Minicourses: **– Xenia de la Ossa: "On the arithmetic and modularity of Calabi-Yau varieties: attractors, periods and counting points over finite fields"**

– Vassily Golyshev: "Hodge conjectures for Calabi-Yau motives: a survey"

Speakers: **Dmitrii Adler, Philip Candelas, Fabien Cléry, Xenia de la Ossa, Maxime Fairon, Veronica Fantini, Ksenia Fedosova, Ilia Gaiur, Vassily Golyshev, Frank Gounelas, Valery Gritsenko, Alexandre Odesski, Casper Oelen, Valentin Ovsienko, Eric Pichon, Igor Reider, Franco Rota, Volodya Roubtsov, Damien Simon, Ronan Terpereau, Pierre Vanhove, Alexandre Veselov**

Laboratoire Paul Painlevé — Faculté des Sciences et Technologies — Université de Lille — C²EMPI Cross Disciplinary Project Lille

Fédération de Recherche Mathématique des Hauts-de-France — Université d'Artois

Schedule

	Monday 26	Tuesday 27	Wednesday 28	Thursday 29	Friday 30
9h30-10h30	Vanhove	de la Ossa 2	Golyshev 2	Ovsienko	Fairon
Coffee break					
11h00-12h00	de la Ossa 1	Terpereau	Fantini	Rota	Candelas
Lunch					
14h00-15h00	Gritsenko	Golyshev 1	Pichon	Gaiur	Roubtsov
Coffee break					
15h30-16h30	Cléry	Gounelas	Veselov	Oelen	Odesski
16h40-17h40	Fedosova	Reider	Adler	Simon	

Titles and abstracts

Dmitrii Adler (Max Planck Institute for Mathematics, Bonn): "Jacobi forms and Kaneko--Zagier type equations"

Abstract: Modular forms and their generalization, Jacobi forms, can be viewed as specific types of special functions. Consequently, it is quite natural to study the differential equations that these functions satisfy. For modular forms, the Kaneko--Zagier equation is a second-order differential equation with respect to the Serre derivative. Analogously, for Jacobi forms, there exists an analogue of the Serre derivative that increases the weight of a Jacobi form by 2 while preserving its index. It is not difficult to describe the kernel of this operator, but finding solutions of even second-order differential equations, known as Kaneko--Zagier type equations, is not such a trivial problem. In my talk, I will present some recent results on this topic.

Philip Candelas (University of Oxford): "L-Functions for Families of Calabi-Yau Varieties"

Abstract: The local zeta functions of a Calabi-Yau manifold are gathered together into a global L - function. These L - functions are conjectured to have modular properties and to satisfy certain functional equations. I will review evidence that supports this, in the context of 1 - parameter families of Calabi-Yau varieties, and will explain the calculation of the conductors for these families and the data structures that describe the L - functions as functions of the parameter. (This is work in progress with X de la Ossa, N Gegelia and D van Straten.)

Fabien Cléry (Loughborough University): **"**Picard modular forms via invariant theory"

Abstract: In previous work with Carel Faber and Gerard van der Geer, we showed how invariant theory can be used to generate all Siegel modular forms of degree 2 and 3. In this talk, we will explain how invariant theory can again be used to construct all Picard modular forms in the case of signature (2,1). This is a joint work with Gerard van der Geer.

Xenia de la Ossa (University of Oxford): "On the arithmetic and modularity of Calabi-Yau varieties: attractors, periods and counting points over finite fields" (minicourse, 2 lectures)

Abstract: The main goal of these talks is to explore questions of common interest for physicists, number theorists and geometers, in the context of Calabi-Yau 3-folds. There are many relations, however we will focus on the rich structure of black hole solutions of superstrings on Calabi-Yau varieties. I will try to give a self contained introduction aimed at a mixed audience of physicists and mathematicians.

The main quantities of interest in the arithmetic context are the number of points of the variety, considered as varieties over finite fields, and how these numbers vary with the parameters of the varieties. The generating function for these numbers is the zeta function, about which much is known in virtue of the Weil conjectures. The first surprise, for a physicist, is that the numbers of these points, and so the zeta function, are given by expressions that involve the periods of the manifold. These same periods determine also many aspects of the physical theory, including the properties of black hole solutions.

I will discuss a number of interesting topics related to the zeta function, the corresponding L-function, and the appearance of modularity for one parameter families of Calabi-Yau manifolds. We will focus on an example for which the quartic numerator of the zeta function of a one parameter family factorises into two quadrics at special values of the parameter. These special values, for which the underlying manifold is smooth, satisfy an algebraic equation with coefficients in $\mathbf{Q}$, so independent of any particular prime. The significance of these factorisations is that they are due to the existence of black hole attractor points in the sense of type II supergravity and are related to a splitting of the Hodge structure and at these special values of the parameter. Modular groups and modular forms arise in relation to these attractor points, in a way that is familiar to mathematicians as a consequence of the Langland's Program, but which is a surprise to a physicist.

The rank two attractor points were found together with Philip Candelas, Mohamed Elmi and Duco van Straten by the application of these number theoretic techniques, and provide the first explicit examples of such attractor points for Calabi-Yau varieties. I will also include some work in progress with Philip Candelas and Eleonora Svanberg on the relation between periods and counting points.

A following up talk by Philip Candelas will discuss in more detail the L-functions and the modularity of one parameter families of Calabi-Yau varieties.

Maxime Fairon (Université de Bourgogne): "Quivers and (supersymmetric) integrable systems"

Veronica Fantini (Université Paris-Saclay): "Local weighted projective spaces and their resurgent invariants"

Abstract. In the framework of the Topological String/Spectral Theory correspondence of Grassi-Hatsuda-Mariño, a new source of invariants (resurgent invariants) for local Calabi-Yau threefolds has been constructed using the theory of resurgence. These invariants are analytic, i.e., they are defined from the analytic continuation of a given function (the first fermionic trace). However, they are conjectured to be related to Gromov-Witten invariants, and they have interesting arithmetic properties.

In this talk, I will focus on the arithmetic properties of the resurgent invariants for local weighted projective spaces, showing that in a few cases, these invariants are coefficients of certain L-functions. This is part of a joint project with C. Rella.

Ksenia Fedosova (Universität Münster): "Convolution identities for divisor functions"

Ilia Gaiur (University of Geneva): "Kernels In-depth: Higher Heun Equations"

Abstract: I will report on joint work in progress with Vasily Golyshev (IHES). Following a program laid out by Golyshev-Mellit-Roubtsov-van Straten, we compute Betti multiplication kernels for the Heun local systems. I will describe ideas standing behind our approach and underline important techniques developed in the framework of this project. In addition, I will show how the analogues of the Heun equations, which we refer to as Higher Heun equations, appear naturally within our framework.

Vassily Golyshev (IHES, Bures-sur-Yvette): "Hodge conjectures for Calabi-Yau motives: a survey" (minicourse, 2 lectures)

Abstract: In the first lecture, I will give a survey of cases where Hodge(-type) conjectures have been proved for Calabi-Yau motives and explain some arithmetic consequences. In the second lecture, I will discuss some challenging unknown cases and present supporting evidence.

Frank Gounelas (Bonn University): "Smooth isotrivial families of curves on K3 surfaces"

Abstract: I will survey recent results with Chen and Dutta regarding existence of smooth curves on K3 surfaces which deform in an isotrivial family. In the particular case of Picard rank one, I will prove these do not occur.

Valery Gritsenko (NRU HSE): "Elliptic genus of Calabi-Yau varieties and modular differential equations"

Vincent Knibbeler (Heriot-Watt University, Edinburgh): "Automorphic Lie algebras on the Riemann sphere" (CANCELLED)

Abstract: After an introduction to automorphic Lie algebras and talking through some elementary but important examples, we discuss the current state of the classification project for the Riemann sphere, and explain why progress halted after the publication in 2017. Then we describe a new approach to the classification problem which allows us to compute automorphic Lie algebras of any simple Lie type.

Alexandre Odesski (Brock University, St. Catharines, Canada): "Explicit formulas for arithmetic support of differential and difference operators"

Abstract: We review the notion of arithmetic support for differential operators in one variable and its analog for q-difference operators. After that we discuss explicit formulas for arithmetic support for certain class of operators in terms of their monodromy data. This is a joint work with Maxim Kontsevich.

Casper Oelen (Heriot-Watt University, Edinburgh): "Elliptic automorphic Lie algebras and integrable systems"

Abstract: Automorphic Lie algebras are a class of infinite-dimensional Lie algebras over the complex numbers that naturally arise in integrable systems, in particular in the context of reduction of Lax pairs. They can be thought of as Lie algebras of meromorphic maps (usually with prescribed poles) from a compact Riemann surface X into a finite-dimensional Lie algebra $\mathfrak{g}$ which are equivariant with respect to a finite group G acting on X and on $\mathfrak{g}$, both by automorphisms. Independently of their origins in integrable systems, they show up in algebra as examples of equivariant map algebras.

In this talk, we will highlight some motivations from integrable systems to study these algebras. We will mainly focus on elliptic automorphic Lie algebras, which, for example, prominently appear in the context of Landau-Lifshitz type of equations. We show that well-known algebras - such as Holod's hidden symmetry algebra of the Landau-Lifshitz equation and the Wahlquist-Estabrook prolongation algebra of the same equation - admit a particularly simple description arising from the automorphicity perspective. They turn out to be isomorphic to a current algebra $\mathfrak{sl}(2, \mathbb{C}) \otimes R$, or to its direct sum with the two-dimensional abelian Lie algebra $\mathbb{C}^2$, in the latter case, where R is a suitable ring of elliptic functions invariant under a particular action of the dihedral group D_2 of order 4. This talk is based on joint work with Sara Lombardo and Vincent Knibbeler.

Valentin Ovsienko (Université de Reims Champagne-Ardenne): "From Catalan numbers to integrable dynamics: continued fractions and Hankel determinants for q-numbers"

Abstract: The classical Catalan and Motzkin numbers have remarkable continued fraction expansions, the corresponding sequences of Hankel determinants consist of -1, 0 and 1 only. We find an infinite family of power series corresponding to q-deformed real numbers that have very similar properties. Moreover, their sequences of Hankel determinants turn out to satisfy Somos and Gale-Robinson recurrences known to be remarkable examples of discrete integrable systems. (Partially based on a joint work with Emmanuel Pedon.)

Eric Pichon (Max Planck institute for Mathematics in Sciences, Leipzig): "Periods of fibre product of elliptic surfaces and the Gamma conjecture"

Abstract: Given two rational elliptic surfaces over the projective line, one may construct a Calabi—Yau threefold by considering their fibre product, following a construction of Schoen. For specific pairs of elliptic surfaces, deforming the parameter of one of the elliptic surfaces in a specific manner one obtains a family of such threefolds called the Hadamard product. These Hadamard products carry a motive of type (1,1,1,1) and the associated Picard-Fuchs equation is a Calabi-Yau operator.

I will describe a method for computing numerical approximations of the periods of such threefolds with very high precision, relying on an explicit description of their homology. This computation allows to probe into the Gamma class formula for these Calabi—Yau operators: we find a formula that seems to fit all example of the Calabi-Yau database.

Igor Reider (Université d'Angers): "Refinement of IVHS invariants: the case of canonical curves". **Abstract:**

Let C be a smooth complex projective curve with canonical divisor K_C very ample. It is well known that the differential of the period map at the point of the moduli space of curves corresponding to C is given by the cup-product

$$H^1(\Theta_C) \longrightarrow H^0(\mathcal{O}_C(K_C))^* \otimes H^1(\mathcal{O}_C)$$

where $\Theta_C = \mathcal{O}_C(-K_C)$ is the holomorphic tangent bundle of C. The cup-product, following Griffiths, stratifies $\mathbb{P}(H^1(\Theta_C))$ by the subvarieties Σ_r according to the rank r of $\xi \in H^1(\Theta_C)$, viewed as the linear map

$$\xi : H^0(\mathcal{O}_C(K_C)) \longrightarrow H^1(\mathcal{O}_C),$$

or, equivalently, by the dimension of the kernel of ξ

$$W_\xi = ker(\xi).$$

The pair (ξ, W_ξ) is one of the invariants of the Infinitesimal Variation of Hodge Structure (IVHS) of Griffiths and his collaborators. I construct a refinement of (ξ, W_ξ) by attaching an intrinsic filtration $W^\bullet_\xi([\phi])$ of W_ξ, varying holomorphically with $[\phi] \in \mathbb{P}(W_\xi)$. The filtration has geometric meaning:

1) it is related to special divisors on C,

2) it 'counts' certain rational normal curves in the canonical embedding of C.

The filtration arises from the skew-symmetric pairing

$$\alpha^{(2)}_\xi : \bigwedge^2 W_\xi \longrightarrow H^0(\mathcal{O}_C(K_C))$$

naturally attached to the IVHS pair (ξ, W_ξ); this could be considered as a 'symplectic' structure on W_ξ.

Conceptually, the refinement concerns the incidence correspondence

$$P = \{([\xi],[\phi]) \in \mathbb{P}(H^1(\Theta_C)) \times \mathbb{P}(H^0(\mathcal{O}_C(K_C)))|\xi\phi = 0 \text{ in } H^1(\mathcal{O}_C)\}$$

which comes with the projections on each factor

$$\begin{tikzcd} & P \arrow[dl, "p_1"'] \arrow[dr, "p_2"] & \\ \mathbb{P}(H^1(\Theta_C)) & & \mathbb{P}(H^0(\mathcal{O}_C(K_C))) \end{tikzcd}$$

The left side of the diagram controls the (infinitesimal) variations of complex structure on C. The refinement exhibits additional structures on the fibres of p_1. In particular, one obtains the stratification of the inverse images $p^{-1}(\Sigma_r)$ of the Griffiths' strata Σ_r into substrata where the length of the filtrations $W^\bullet_\xi(\phi)$, $([\xi],[\phi]) \in p^{-1}(\Sigma_r)$, remains constant. On each such substratum new aspects emerge:

- quiver representations,
- Fano toric variety with a distinguished anti-canonical divisor,
- dimer models.

The quiver emerges from the construction and properties of the refinement; the Fano toric variety arises formally from the graph underling the quiver, but it also has a meaning of moduli of a sort of Higgs structures of the linear algebra of the refinement. The graph underlying the refinement becomes an important part of the theory: it connects to topics such as the Topological Quantum field theory, moduli spaces of elliptic curves with marked points, modular curves, higher categorical structures. In my talk I will try to give an idea how all of the above fit together.

Franco Rota (Université Paris-Saclay): "Towards Homological Mirror Symmetry for log del Pezzo surfaces"

Abstract: Motivated by Homological Mirror Symmetry, we study a series of singular surfaces called log del Pezzo. I will describe the derived category of a series of log del Pezzo's, using the McKay correspondence and explicit birational geometry. If time permits, I'll mention early mirror results, focusing mostly on the special case of

smooth low degree del Pezzo surfaces. For these, we will compare several mirror constructions using the language of pseudolattices. This is joint work with Giulia Gugiatti.

Volodya Roubtsov (Université d'Angers): "Bessel and beyond (old songs with new motives)"

Damien Simon (Université Paris-Saclay): "Chiral differential operators on a reductive group and representation theory"

Abstract: Vertex algebras of chiral differential operators on a complex reductive group G are "Kac-Moody" versions of the usual algebra of differential operators on G. Their categories of modules are especially interesting because they are related to the theory of algebraic D-modules on the loop group of G. That allows one to reformulate some conjectures of the (quantum) geometric Langlands program in the language of vertex algebras. For instance, in view of the geometric Satake equivalence, one may expect the appearance of the category of representations of the Langlands dual group of G. I will explain why this is a reasonable expectation and give some basic statements.

Ronan Terpereau (Université de Lille): "Maximal connected algebraic subgroups of the real Cremona group"

Abstract. In this talk, we will explain how we were able to achieve a (partial?) classification of the maximal connected algebraic subgroups of the real Cremona group in rank 3 by determining the rational real forms of certain complex Mori fiber spaces. This work was carried out in collaboration with Susanna Zimmermann.

Pierre Vanhove (IPhT CEA & CERN): "Motives for Feynman integrals"

Abstract. We explain how to attach a motive to a Feynman diagram, and present a classification of motives that appear depending on the number of loops of the graphs. We show how to connect this classification with the understanding of Feynman integrals as relative periods of twisted cohomology.

Alexandre Veselov (Univ. of Loughborough, UK): "Markov fractions and the slopes of the exceptional bundles on $\mathbf{P}^2$"

Abstract. We show that the Markov fractions introduced recently by Boris Springborn are precisely the slopes of the exceptional vector bundles on $\mathbf{P}^2$ studied in 1980s by Drézet and Le Potier and by Rudakov. In particular, we provide a simpler proof of Rudakov's result claiming that the ranks of the exceptional bundles on $\mathbf{P}^2$ are Markov numbers.