Algebraic geometry, integrable systems and automorphic forms

May 26-30, 2025

Lille (France)

Laboratoire Paul Painlevé

Salle de réunions, bâtiment M2

Scientific Committee: Ekaterina Amerik, Valery Gritsenko, Anne Moreau, Pierre Vanhove

Organizers: Abdelghani El Mazouni,Fatima Laytimi, Dimitri Markushevich

Minicourses: – **Xenia de la Ossa: "Attractor mechanism and arithmetic of CY manifolds"**

– Vassily Golyshev: "Hodge conjectures for Calabi-Yau motives: a survey"

Speakers: **Dmitrii Adler, Fabien Cléry, Xenia de la Ossa, Maxime Fairon, Veronica Fantini, Ksenia Fedosova, Ilia Gaiur, Vassily Golyshev, Frank Gounelas, Valery Gritsenko, Kenji Iohara, Vincent Knibbeler, Dimitri Markushevich, Alexandre Odesski, Casper Oelen, Valentin Ovsienko, Eric Pichon, Igor Reider, Franco Rota, Volodya Roubtsov, Pierre Vanhove, Alexander Veselov**

Schedule

	Monday 26	Tuesday 27	Wednesday 28	Thursday 29	Friday 30
9h30-10h30	Vanhove	de la Ossa 2	Golyshev 2	Ovsienko	Fairon
Coffee break					
11h00-12h00	de la Ossa 1	Golyshev 1	Fantini	Rota	Markushevich
Lunch					
14h00-15h00	Gritsenko	Iohara	Pichon	Gaiur	Roubtsov
Coffee break					
15h30-16h30	Cléry	Gounelas	Veselov	Oelen	Odesski
16h40-17h40	Knibbeler	Reider	Adler	Fedosova	

Titles and abstracts

Dmitrii Adler (Max Planck Institute for Mathematics, Bonn): "Jacobi forms and Kaneko--Zagier type equations"

Abstract: Modular forms and their generalization, Jacobi forms, can be viewed as specific types of special functions. Consequently, it is quite natural to study the differential equations that these functions satisfy. For modular forms, the Kaneko--Zagier equation is a second-order differential equation with respect to the Serre derivative. Analogously, for Jacobi forms, there exists an analogue of the Serre derivative that increases the weight of a Jacobi form by 2 while preserving its index. It is not difficult to describe the kernel of this operator, but finding solutions of even second-order differential equations, known as Kaneko--Zagier type equations, is not such a trivial problem. In my talk, I will present some recent results on this topic.

Katia Amerik (Université Paris-Saclay): **TBA**

Fabien Cléry (Loughborough University): "Picard modular forms via invariant theory"

Abstract: In previous work with Carel Faber and Gerard van der Geer, we showed how invariant theory can be used to generate all Siegel modular forms of degree 2 and 3. In this talk, we will explain how invariant theory can again be used to construct all Picard modular forms in the case of signature (2,1). This is a joint work with Gerard van der Geer.

Xenia de la Ossa (University of Oxford): "Attractor mechanism and arithmetic of CY manifolds" (minicourse, 2 lectures)

Abstract: The minicourse involves Hodge structures, periods, zeta-functions and modular forms arising from families of Calabi--Yau manifolds and their relation to supergravity and string theory.

Maxime Fairon (Université de Bourgogne): "Quivers and (supersymmetric) integrable systems"

Veronica Fantini (Université Paris-Saclay): **TBA**

Ksenia Fedosova (Universität Münster): "Convolution identities for divisor functions"

Ilia Gaiur (University of Geneva): "Kernels In-depth: Higher Heun Equations"

Abstract: I will report on joint work in progress with Vasily Golyshev (IHES). Following a program laid out by Golyshev-Mellit-Roubtsov-van Straten, we compute Betti multiplication kernels for the Heun local systems. I will describe ideas standing behind our approach and underline important techniques developed in the framework of this project. In addition, I will show how the analogues of the Heun equations, which we refer to as Higher Heun equations, appear naturally within our framework.

Vassily Golyshev (IHES, Bures-sur-Yvette): "Hodge conjectures for Calabi-Yau motives: a survey" (minicourse, 2 lectures)

Abstract: In the first lecture, I will give a survey of cases where Hodge(-type) conjectures have been proved for Calabi-Yau motives and explain some arithmetic consequences. In the second lecture, I will discuss some challenging unknown cases and present supporting evidence.

Frank Gounelas (Bonn University): "Smooth isotrivial families of curves on K3 surfaces"

Abstract: I will survey recent results with Chen and Dutta regarding existence of smooth curves on K3 surfaces which deform in an isotrivial family. In the particular case of Picard rank one, I will prove these do not occur.

Valery Gritsenko (NRU HSE): "Elliptic genus of Calabi-Yau varieties and modular differential equations"

Kenji Iohara (Université Claude Bernard Lyon 1): "Modular Invariance of the characters of certain affine Lie algebras"

Abstract: We are revisiting an old paper by Macdonald (1972). It turns out that Macdonald calculated several evaluations of denominator identity to obtain several eta-produt identities. Inspired by this work, we have studied its implications at the level of the character, and we found that for $A_{2l}^{(2)}$ type affine Lie algebra, its characters and super-characters span a vector space admitting a $\mathrm{SL}_2(\mathbf{Z})$-action. We are also studying another case evoked by Macdonald. I will report on the state of art of this computation.

Vincent Knibbeler (Heriot-Watt University, Edinburgh): "Automorphic Lie algebras on the Riemann sphere"

Alexandre Odesski (Brock University, St. Catharines, Canada): **TBA**

Casper Oelen (Heriot-Watt University, Edinburgh): "Elliptic automorphic Lie algebras and integrable systems"

Abstract: Automorphic Lie algebras are a class of infinite-dimensional Lie algebras over the complex numbers that naturally arise in integrable systems, in particular in the context of reduction of Lax pairs. They can be thought of as Lie algebras of meromorphic maps (usually with prescribed poles) from a compact Riemann surface X into a finite-dimensional Lie algebra $\mathfrak{g}$ which are equivariant with respect to a finite group G acting on X and on $\mathfrak{g}$, both by automorphisms. Independently of their origins in integrable systems, they show up in algebra as examples of equivariant map algebras.

In this talk, we will highlight some motivations from integrable systems to study these algebras. We will mainly focus on elliptic automorphic Lie algebras, which, for example, prominently appear in the context of Landau-Lifshitz type of equations. We show that well-known algebras - such as Holod's hidden symmetry algebra of the Landau-Lifshitz equation and the Wahlquist-Estabrook prolongation algebra of the same equation - admit a particularly simple description arising from the automorphicity perspective. They turn out to be isomorphic to a current algebra $\mathfrak{sl}(2, \mathbb{C}) \otimes R$, or to its direct sum with the two-dimensional abelian Lie algebra $\mathbb{C}^2$, in the latter case, where R is a suitable ring of elliptic functions invariant under a particular action of the dihedral group D_2 of order 4. This talk is based on joint work with Sara Lombardo and Vincent Knibbeler.

Valentin Ovsienko (Université de Reims Champagne-Ardenne): "From Catalan numbers to integrable dynamics: continued fractions and Hankel determinants for q-numbers"

Abstract: The classical Catalan and Motzkin numbers have remarkable continued fraction expansions, the corresponding sequences of Hankel

determinants consist of -1, 0 and 1 only. We find an infinite family of power series corresponding to q-deformed real numbers that have very similar properties. Moreover, their sequences of Hankel determinants turn out to satisfy Somos and Gale-Robinson recurrences known to be remarkable examples of discrete integrable systems. (Partially based on a joint work with Emmanuel Pedon.)

Eric Pichon (Max Planck institute for Mathematics in Sciences, Leipzig): **TBA**

Igor Reider (Université d'Angers): "Refinement of IVHS invariants: the case of canonical curves". **Abstract:**

Let C be a smooth complex projective curve with canonical divisor K_C very ample. It is well known that the differential of the period map at the point of the moduli space of curves corresponding to C is given by the cup-product

$$H^1(\Theta_C) \longrightarrow H^0(\mathcal{O}_C(K_C))^* \otimes H^1(\mathcal{O}_C)$$

where $\Theta_C = \mathcal{O}_C(-K_C)$ is the holomorphic tangent bundle of C. The cup-product, following Griffiths, stratifies $\mathbb{P}(H^1(\Theta_C))$ by the subvarieties Σ_r according to the rank r of $\xi \in H^1(\Theta_C)$, viewed as the linear map

$$\xi : H^0(\mathcal{O}_C(K_C)) \longrightarrow H^1(\mathcal{O}_C),$$

or, equivalently, by the dimension of the kernel of ξ

$$W_\xi = ker(\xi).$$

The pair (ξ, W_ξ) is one of the invariants of the Infinitesimal Variation of Hodge Structure (IVHS) of Griffiths and his collaborators. I construct a refinement of (ξ, W_ξ) by attaching an intrinsic filtration $W_\xi^\bullet([\phi])$ of W_ξ, varying holomorphically with $[\phi] \in \mathbb{P}(W_\xi)$. The filtration has geometric meaning:

1) it is related to special divisors on C,

2) it 'counts' certain rational normal curves in the canonical embedding of C.

The filtration arises from the skew-symmetric pairing

$$\alpha_\xi^{(2)} : \bigwedge^2 W_\xi \longrightarrow H^0(\mathcal{O}_C(K_C))$$

naturally attached to the IVHS pair (ξ, W_ξ); this could be considered as a 'symplectic' structure on W_ξ.

Conceptually, the refinement concerns the incidence correspondence

$$P = \{([\xi], [\phi]) \in \mathbb{P}(H^1(\Theta_C)) \times \mathbb{P}(H^0(\mathcal{O}_C(K_C))) | \xi\phi = 0 \text{ in } H^1(\mathcal{O}_C)\}$$

which comes with the projections on each factor

$$\mathbb{P}(H^1(\Theta_C)) \xleftarrow{p_1} P \xrightarrow{p_2} \mathbb{P}(H^0(\mathcal{O}_C(K_C)))$$

The left side of the diagram controls the (infinitesimal) variations of complex structure on C. The refinement exhibits additional structures on the fibres of p_1. In particular, one obtains the stratification of the inverse images $p^{-1}(\Sigma_r)$ of the Griffiths' strata Σ_r into substrata where the length of the filtrations $W^{\bullet}_{\xi}(\phi)$, $([\xi],[\phi]) \in p^{-1}(\Sigma_r)$, remains constant. On each such substratum new aspects emerge:

- quiver representations,
- Fano toric variety with a distinguished anti-canonical divisor,
- dimer models.

The quiver emerges from the construction and properties of the refinement; the Fano toric variety arises formally from the graph underling the quiver, but it also has a meaning of moduli of a sort of Higgs structures of the linear algebra of the refinement. The graph underlying the refinement becomes an important part of the theory: it connects to topics such as the Topological Quantum field theory, moduli spaces of elliptic curves with marked points, modular curves, higher categorical structures. In my talk I will try to give an idea how all of the above fit together.

Franco Rota (Université Paris-Saclay): "Towards Homological Mirror Symmetry for log del Pezzo surfaces"

Abstract: Motivated by Homological Mirror Symmetry, we study a series of singular surfaces called log del Pezzo. I will describe the derived category of a series of log del Pezzo's, using the McKay correspondence and explicit birational geometry. If time permits, I'll mention early mirror results, focusing mostly on the special case of smooth low degree del Pezzo surfaces. For these, we will compare several mirror constructions using the language of pseudolattices. This is joint work with Giulia Gugiatti.

Volodya Roubtsov (Université d'Angers): **TBA**

Pierre Vanhove (IPhT CEA & CERN): "Motives for Feynman integrals"

Abstract. We explain how to attach a motive to a Feynman diagram, and present a classification of motives that appear depending on the number of loops of the graphs. We show how to connect this classification with the understanding of Feynman integrals as relative periods of twisted cohomology.

Alexander Veselov (Loughborough University): **TBA**